INVESTIGATION 4-B

Examining Collisions

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

You have learned the definition of elastic and inelastic collisions, but are there characteristics that allow you to predict whether a collision will be elastic? In this investigation, you will observe and analyze several collisions and draw conclusions regarding whether a type of collision will be elastic or inelastic.

Problem

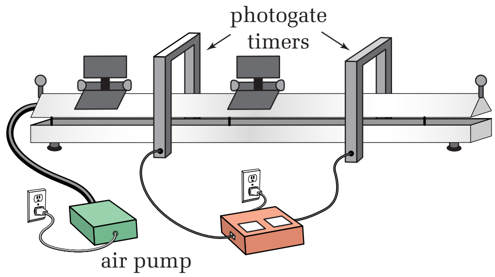
What are the characteristics of elastic and inelastic collisions?

Equipment

- air track (with source of compressed air)
- 2 gliders (identical, either middle- or large-sized)
- 2 photogate timers
- laboratory balance
- 4 glider bumper springs
- 2 Velcro™ bumpers (or a needle and a piece of wax)
- 2 velocity flags (10 cm) (or file cards cut to a 10 cm length)
- modelling clay

Procedure

1. Set up the air track and adjust the levelling screw to ensure that the track is horizontal. You can test whether the track is level by turning on the air pressure and placing a glider on the track. Hold the glider still and then release it. If the track is level, the glider will remain in place. If the glider gradually starts moving, the air track is not level.
2. Attach a velocity flag (or 10 cm card) and two bumper springs to each glider. If only one bumper spring is attached, the glider might not be properly balanced.
3. Position the photogates about one fourth the length of the track from each end, as shown in the diagram. Adjust the height of the photogates so that the velocity flags will pass through the gates smoothly but will trigger the gates.



4. Label one glider "A" and the other glider "B." Use the laboratory balance to determine accurately the mass of each glider.
5. With the air flowing, place glider A on the left end of the air track and glider B in the centre.
6. Perform a test run by pushing glider A so that it collides with glider B. Ensure that the photogates are placed properly so that the flags are not inside the gates when the gliders are in contact. Adjust the positions of the photogates, if necessary.
7. The first set of trials will be like the test run, with glider A on the left end of the track and glider B in the centre. Turn on the photogates and press the reset button. Push glider A and allow it to collide with glider B. Allow both gliders to pass through a photogate after the collision, then catch them before they bounce back and pass through a photogate again. Record the data in a table similar to the one shown on the next page. Since all of the motion will be in one dimension, only positive and negative signs will be needed to indicate direction. Vector notations will not be necessary.
The displacement, Δd, is the distance that the gliders travelled while passing through the photogates. This displacement is the length of the flag. Time Δt_i is the time that a glider spent in the photogate before the

collision, while Δt_f is the time the glider took to pass through the photogate after the collision. Calculate velocity, v, from the displacement and the time interval. Be sure to include positive and negative signs.

	Glider A (mass = ?)				
Trial	$\Delta\vec{d}$	Δt_i	$\vec{v}_i$	Δt_f	$\vec{v}_f$
1					
2					
3					

	Glider B (mass = ?)		
Trial	$\Delta\vec{d}$	Δt_f	$\vec{v}_f$
1			
2			
3			

8. Increase the mass of glider B by attaching some modelling clay to it. Be sure that the clay is evenly distributed along the glider. If the glider tips to the side or to the front or back, the motion will not be smooth. Determine the mass of glider B. Repeat step 7 for the two gliders, which are now of unequal mass.

9. Exchange the gliders and their labels. That is, the glider with the extra mass is on the left and becomes glider "A." The glider with no extra mass should be in the centre and labelled "B." Repeat step 7 for the new arrangement of gliders.

10. Remove the clay from the glider. Place one glider at each end of the track. Practise starting both gliders at the same time, so that they collide near the middle of the track. The collision must not take place while either glider is in a photogate. When you have demonstrated that you can carry out the collision correctly, perform three trials and record the data in a table similar to the ones shown here. This table will need two additional columns — one for the initial time for glider B and a second for the initial velocity of glider B.

11. Remove the bumper springs from one end of each glider and attach the Velcro™ bumpers. (If you do not have Velcro™ bumpers, you can attach a large needle to one glider and a piece of wax to the other. Test to ensure that the needle will hit the wax when the gliders collide.)

12. With the Velcro™ bumpers attached, perform three sets of trials similar to those in steps 7, 8, and 9. You might need to perform trial runs and adjust the position of the photogates so that both gliders can pass through the photogate before reaching the right-hand end of the air track. Record the data in tables similar to those you used previously.

Analyze and Conclude

1. For each glider in each trial, calculate the initial momentum (before the collision) and the final momentum (after the collision).

2. For each trial, calculate the total momentum of both gliders before the collision and the total momentum of both after the collision.

3. For each trial, compare the momentum before and after the collision. Describe how well the collisions demonstrated conservation of momentum.

4. In any case for which momentum did not seem to be conserved, provide possible explanations for errors.

5. Calculate the kinetic energy of each glider in each trial. Then calculate the total kinetic energy of both gliders before and after the collision for each trial.

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6. Compare the kinetic energies before and after the collisions and decide which collisions were elastic and which were inelastic. Due to measurement errors, do not expect the kinetic energies to be identical before and after a collision. Decide if the values appear to be close enough that the differences could be attributed to measurement errors.
7. Examine the nature of the collisions that you considered to be elastic and those that you classed as inelastic. Look for a trend that would permit you to predict whether a collision would be elastic or inelastic. Discuss your conclusions with the rest of class. How well did your conclusions agree with those of other class members?

Apply and Extend

8. If you have access to an air table, a strobe light, and a Polaroid™ camera, you can observe and collect data for collisions in two dimensions. (If you do not have the equipment, your teacher might be able to provide you with simulated photographs.) Using the laboratory balance, determine the mass of each of two pucks. If the pucks have nearly the same mass, add some mass to one of them, using modelling clay.

CAUTION People with certain medical conditions, such as epilepsy, can experience seizures if exposed to strobe lighting.

9. With the air pressure on, place a puck in the centre of the table. Direct the strobe light onto the table and set up the camera so that it is above the table and pointing down. Turn on the strobe light and set the camera for a long exposure time. At the moment that one partner pushes the other puck toward the stationary puck in the centre of the table, the other partner should take a picture. Take enough photographs to provide each pair of partners with a photograph.
10. Determine the scale of the photograph by determining the ratio of the size of the air table to its apparent size in the photograph. Measure two or three distances before and after the collision. Correct the distances by using the scale that you determined. Measure the angles that the pucks took after the collision in relation to the original direction.
11. Using the rate at which the strobe light was flashing, determine the time between flashes. Calculate the velocity, momentum, and kinetic energy of each puck before and after the collision.
12. Compare the total momentum before and after the collision and comment on how well the motion seemed to obey the law of conservation of momentum.
13. Compare the total kinetic energies before and after the collision. Decide whether the collisions were elastic or nearly so.
14. Compare your results from your one-dimensional data and two-dimensional data, and comment on any differences that you noticed.
15. Review the results you obtained in Investigation 4-A, Newton's Cradle. Do you think the collisions were elastic or inelastic? Explain why.
16. Support your answer to question 15 by performing trial calculations. Assume that the spheres in Newton's cradle each has a mass of 0.200 kg and that, when one sphere collided with the row, it was moving at 0.100 m/s. Imagine that, when one sphere collided with the row, two spheres bounced up from the opposite end. Calculate the velocity that the two spheres would have to have in order to conserve momentum. Calculate the kinetic energy before and after. Use these calculations to explain why you did not observe certain patterns in the motion of Newton's cradle.

SAMPLE PROBLEM

Classifying a Collision

A 0.0520 kg golf ball is moving east with a velocity of 2.10 m/s when it collides, head on, with a 0.155 kg billiard ball. If the golf ball rolls directly backward with a velocity of –1.04 m/s, was the collision elastic?

Conceptualize the Problem

- *Momentum* is always conserved in a collision.
- If the collision is *elastic*, kinetic energy must also be conserved.
- The motion is in one dimension, so only positive and negative signs are necessary to indicate directions.

Identify the Goal

Is the total kinetic energy of the system before the collision, E_{kg}, equal to the total kinetic of the system after the collision, $E'_{kg} + E'_{kb}$?

Identify the Variables and Constants

Known		Implied	Unknown	
$m_g = 0.0520\ \text{kg}$	$v_g = +2.10\ \frac{\text{m}}{\text{s}}$	$v_b = 0.0\ \frac{\text{m}}{\text{s}}$	v'_b	E'_{kg}
$m_b = 0.155\ \text{kg}$	$v'_g = -1.04\ \frac{\text{m}}{\text{s}}$		E_{kg}	E'_{kb}

Develop a Strategy

Since momentum is always conserved, use the law of conservation of momentum to find the velocity of the billiard ball after the collision.

$$m_g v_g + m_b v_b = m_g v'_g + m_b v'_b$$
$$m_g v_g + 0.0 - m_g v'_g = m_b v'_b$$
$$v'_b = \frac{m_g v_g - m_g v'_g}{m_b}$$
$$v'_b = \frac{(0.0520\ \text{kg})(2.10\ \frac{\text{m}}{\text{s}}) - (0.0520\ \text{kg})(-1.04\ \frac{\text{m}}{\text{s}})}{0.155\ \text{kg}}$$
$$v'_b = 1.0534\ \frac{\text{m}}{\text{s}}$$

Calculate the kinetic energy of the golf ball before the collision.

$$E_{kg} = \frac{1}{2} m_g v_g^2$$
$$E_{kg} = \frac{1}{2}(0.0520\ \text{kg})\left(2.10\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E_{kg} = 0.114\ 66\ \text{J}$$

Calculate the sum of the kinetic energies of the balls after the collision.

$$E'_{kg} = \frac{1}{2} m_g v'^2_g$$
$$E'_{kg} = \frac{1}{2}(0.0520\ \text{kg})\left(-1.04\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E'_{kg} = 0.028\ 12\ \text{J}$$
$$E'_{kb} = \frac{1}{2} m_b v'^2_b$$
$$E'_{kb} = \frac{1}{2}(0.155\ \text{kg})\left(1.0534\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E'_{kb} = 0.086\ 00\ \text{J}$$
$$E'_{kg} + E'_{kb} = 0.028\ 12\ \text{J} + 0.085\ 99\ \text{J}$$
$$E'_{kg} + E'_{kb} = 0.114\ 12\ \text{J}$$

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The kinetic energies before and after the collision are the same to the third decimal place. Therefore, the collision was probably elastic.

Validate the Solution

Although the kinetic energies before and after the collision differ in the fourth decimal place, the difference is less than 1%. Since the data contained only three significant digits, this difference could easily be due to the precision of the measurement. Therefore, it is fair to say that the collision was elastic.

PRACTICE PROBLEMS

16. A billiard ball of mass 0.155 kg moves with a velocity of 12.5 m/s toward a stationary billiard ball of identical mass and strikes it with a glancing blow. The first billiard ball moves off at an angle of 29.7˚ clockwise from its original direction, with a velocity of 9.56 m/s. Determine whether the collision was elastic.

17. Car A, with a mass of 1735 kg, was travelling north at 45.5 km/h and Car B, with a mass of 2540 kg, was travelling west at 37.7 km/h when they collided at an intersection. If the cars stuck together after the collision, what was their combined momentum? Was the collision elastic or inelastic?

Elastic Collisions

By now, you have probably concluded that when objects collide, become deformed, and stick together, the collision is inelastic. Physicists say that such a collision is *completely inelastic*. Conversely, when hard objects such as billiard balls collide, bounce off each other, and return to their original shape, they have undergone elastic collisions. Very few collisions are perfectly elastic, but in many cases, the loss of kinetic energy is so small that it can be neglected.

Since both kinetic energy and momentum are conserved in perfectly elastic collisions, as many as four independent equations can be used to solve problems. Since you have two equations, you can solve for up to four unknown quantities. When combining these equations, however, the math becomes quite complex for all cases except head-on collisions, for which all motion is in one dimension.

An analysis of head-on collisions yields some very informative results, however. For example, if you know the velocities of the two masses before a collision, you can determine what the velocities will be after the collision. The following derivation applies to a mass, m_1, that is rolling toward a stationary mass, m_2. Follow the steps to find the velocities of the two objects after the collision in terms of their masses and the velocity of the first mass before the collision. Since the motion in head-on collisions is in one dimension, vector notations will not be used.

LANGUAGE LINK

In science, the word “elastic” does not mean “easily stretched.” In fact, it can mean exactly the opposite. For example, glass is very elastic, up to its breaking point. Also, “elastic” is the opposite of “plastic.” Find the correct meanings of the words “elastic” and “plastic” and then explain why “elastic” is an appropriate term to apply to collisions in which kinetic energy is conserved.

- Write the equations for the conservation of momentum and kinetic energy for a perfectly elastic collision, inserting zero for the velocity of the second mass before the collision.

$$m_1v_1 + 0 = m_1v_1' + m_2v_2'$$
$$\frac{1}{2}m_1v_1^2 + 0 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

- Multiply by 2 both sides of the equation for conservation of kinetic energy.

$$m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2$$

- Algebraically rearrange both equations so that terms describing mass 1 are on the left-hand side of the equations and terms describing mass 2 are on the right-hand side.

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2$$

- Factor m_1 out of the left-hand side of both equations.

$$m_1(v_1 - v_1') = m_2v_2'$$
$$m_1(v_1^2 - v_1'^2) = m_2v_2'^2$$

- Divide the first equation by the second equation.

$$\frac{\cancel{m_1}(v_1 - v_1')}{\cancel{m_1}(v_1^2 - v_1'^2)} = \frac{\cancel{m_2}v_2'}{\cancel{m_2}v_2'^2}$$

- Notice that the masses cancel. Expand the expression in the denominator on the left. Notice that it is the difference of perfect squares.

$$\frac{\cancel{(v_1 - v_1')}}{\cancel{(v_1 - v_1')}(v_1 + v_1')} = \frac{\cancel{v_2'}}{v_2'^{\cancel{2}}}$$

- Simplify. Solve the equation for v_2' by inverting. Also, solve the equation for v_1'.

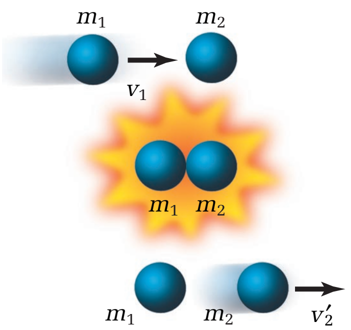
$$\frac{1}{(v_1 + v_1')} = \frac{1}{v_2'}$$
$$v_2' = v_1 + v_1'$$
$$v_1' = v_2' - v_1$$

- Develop two separate equations by substituting the values for v_1' and v_2' above into the equation for conservation of momentum, $m_1v_1 - m_1v_1' = m_2v_2'$. Expand and rearrange the equations and then solve for v_1' (left) and v_2' (right).

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1v_1' = m_2(v_1 + v_1')$$
$$m_1v_1 - m_1v_1' = m_2v_1 + m_2v_1'$$
$$m_1v_1' + m_2v_1' = m_1v_1 - m_2v_1$$
$$v_1'(m_1 + m_2) = v_1(m_1 - m_2)$$
$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$$

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1(v_2' - v_1) = m_2v_2'$$
$$m_1v_1 - m_1v_2' + m_1v_1 = m_2v_2'$$
$$2m_1v_1 = m_1v_2' + m_2v_2'$$
$$2m_1v_1 = (m_1 + m_2)v_2'$$
$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

The two equations derived above allow you to find the velocities of two masses after a head-on collision in which a moving mass collides with a stationary mass. Without doing any calculations, however, you can draw some general conclusions. First, consider the case in which the two masses are identical.



When one moving mass collides head on with an identical stationary mass, the first mass stops. The second mass then moves with a velocity identical to the original velocity of the first mass.

Case 1: $m_1 = m_2$

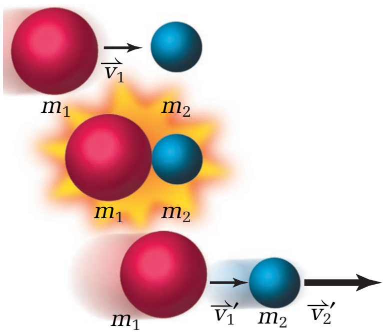
Since the masses are equal, call them both "m." Substitute m into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' = \left(\frac{m - m}{m + m}\right) v_1 \qquad v_2' = \left(\frac{2m}{m + m}\right) v_1$$

$$v_1' = \left(\frac{0}{m + m}\right) v_1 \qquad v_2' = \left(\frac{\cancel{2m}}{\cancel{2m}}\right) v_1$$

$$v_1' = 0 \qquad v_2' = v_1$$



When one moving mass collides head on with a much smaller stationary mass, the first mass continues at nearly the same speed. The second mass then moves with a velocity that is approximately twice the original velocity of the first mass.

Case 2: $m_1 >>> m_2$

Since mass 1 is much larger than mass 2, you can almost ignore the mass of the second object in your calculations. You can therefore make the following approximations.

$$m_1 - m_2 \cong m_1 \quad \text{and} \quad m_1 + m_2 \cong m_1$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{\cancel{m_1}}{\cancel{m_1}}\right) v_1 \qquad v_2' \cong \left(\frac{2\cancel{m_1}}{\cancel{m_1}}\right) v_1$$

$$v_1' \cong v_1 \qquad v_2' \cong 2v_1$$

Case 3: $m_1 <<< m_2$

Since mass 1 is much smaller than mass 2, you can ignore the mass of the first object in your calculations. You can therefore make the following approximations.

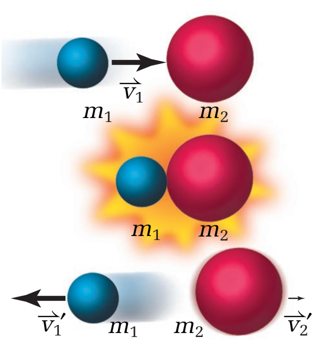
$$m_1 - m_2 \cong -m_2 \quad \text{and} \quad m_1 + m_2 \cong m_2 \quad \text{and} \quad m_1 \cong 0$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{-\cancel{m_2}}{\cancel{m_2}}\right) v_1 \qquad v_2' \cong \left(\frac{0}{m_2}\right) v_1$$

$$v_1' \cong -v_1 \qquad v_2' \cong 0$$



When one moving mass collides head on with a much larger stationary mass, the first mass bounces backward with a velocity opposite in direction and almost the same in magnitude as its original velocity. The motion of the second mass is almost imperceptible.

Conceptual Problems

- Using the special cases of elastic collisions, qualitatively explain what would happen in each of the following situations.
  (a) A bowling ball collides head on with a single bowling pin.
  (b) A golf ball hits a tree.
  (c) A marble collides head on with another marble that is not moving.
- Cars, trucks, and motorcycles do not undergo elastic collisions, but the general trend of the motion is similar to the motion of objects involved in elastic collisions. Describe, in very general terms, what would happen in each of the following cases. In each case, assume that the vehicles did not become attached to each other.
  (a) A very small car runs into the back of a parked tractor-trailer.
  (b) A mid-sized car runs into the back of another mid-sized car that has stopped at a traffic light.
  (c) A pickup truck runs into a parked motorcycle.

ELECTRONIC LEARNING PARTNER

Study the effects of the variable elasticity of a bouncing ball by using the interactive activity in your Electronic Learning Partner.

Inelastic Collisions

When you are working with inelastic collisions, you can apply only the law of conservation of momentum to the motion of the objects at the instant of the collision. Depending on the situation, however, you might be able to apply the laws of conservation of energy to motion just before or just after the collision. For example, a ballistic pendulum can be used to measure the velocity of a projectile such as a bullet, as illustrated in Figure 4.13. When the bullet collides with the wooden block of the ballistic pendulum, it becomes embedded in the wood, making the collision completely inelastic.

After the collision, you can apply the law of conservation of mechanical energy to the motion of the pendulum. The kinetic energy of the pendulum at the instant after the collision is converted into potential energy of the pendulum bob. By measuring the height to which the pendulum rises, you can calculate the velocity of the bullet just before it hit the pendulum, as shown in the following sample problem.

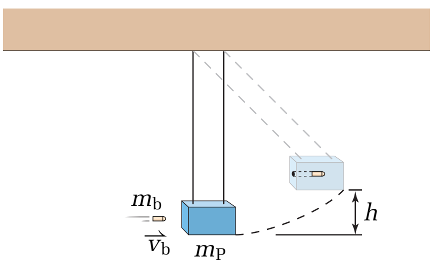


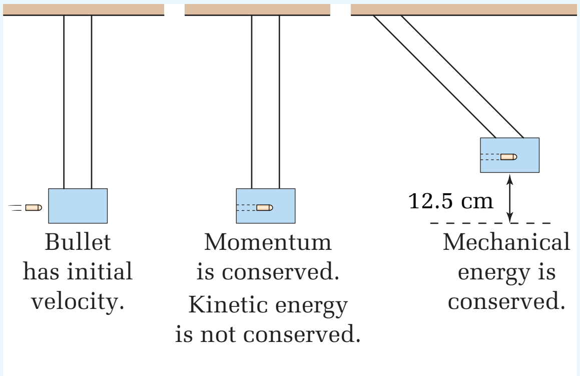
Figure 4.13 A ballistic pendulum is designed to have as little friction as possible. Therefore, you can assume that, at the top of its swing, the gravitational potential energy of the pendulum bob is equal to the kinetic energy of the pendulum bob at the lowest point of its motion.

SAMPLE PROBLEMS

Energy Conservation Before and After a Collision

1. **A forensic expert needed to find the velocity of a bullet fired from a gun in order to predict the trajectory of a bullet. He fired a 5.50 g bullet into a ballistic pendulum with a bob that had of mass 1.75 kg. The pendulum swung to a height of 12.5 cm above its rest position before dropping back down. What was the velocity of the bullet just before it hit and became embedded in the pendulum bob?**

Conceptualize the Problem



- Sketch the positions of the bullet and pendulum bob just before the collision, just after the collision, and with the pendulum at its highest point.
- When the bullet hit the pendulum, *momentum* was *conserved*.
- If you can find the *velocity* of the combined bullet and pendulum bob after the collision, you can use conservation of momentum to find the *velocity* of the bullet before the collision.
- The collision was completely inelastic so *kinetic energy* was *not* conserved.
- However, you can assume that the friction of the pendulum is negligible, so *mechanical energy* of the pendulum was *conserved*.
- The *gravitational potential energy* of the combined masses at the highest point of the pendulum is equal to the *kinetic energy* of the combined masses at the lowest point of the pendulum.
- If you know the kinetic energy of the combined masses just after the collision, you can find the *velocity* of the masses just *after* the collision.
- Use the subscripts "b" for the bullet and "p" for the pendulum.

Identify the Goal

The velocity, v_b, of the bullet just before it hit the ballistic pendulum

Identify the Variables and Constants

Known		Implied	Unknown	
$m_b = 5.50$ g	$\Delta h = 12.5$ cm	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{v}_b$	E_g
$m_p = 1.75$ kg			$\vec{v}_p$	E_k

Develop a Strategy

To find the velocity of the combined masses of the bullet and pendulum bob just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

$$E_{k(\text{bottom})} = E_{g(\text{top})}$$

Substitute the expressions for kinetic energy and gravitational potential energy that you learned in previous physics courses. Solve for velocity. Convert all units to SI units.

$$\frac{1}{2}\cancel{m}v_{\text{bottom}}^2 = \cancel{m}g\Delta h$$

$$v_{\text{bottom}}^2 = 2g\Delta h$$

$$v_{\text{bottom}} = \sqrt{2g\Delta h}$$

$$v_{\text{bottom}} = \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(12.5\ \cancel{\text{cm}})\left(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)}$$

Define the direction of the bullet as positive during and immediately after the collision.

$$v_{\text{bottom}} = \sqrt{2.4525\ \frac{\text{m}^2}{\text{s}^2}}$$

$$v_{\text{bottom}} = \pm 1.566\ \frac{\text{m}}{\text{s}}$$

Apply the conservation of momentum to find the velocity of the bullet before the collision. Convert all units to SI units.

$$m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{p}}\vec{v}_{\text{p}} = m_{\text{b}}\vec{v}\,'_{\text{b}} + m_{\text{p}}\vec{v}\,'_{\text{p}}$$

$$m_{\text{b}}\vec{v}_{\text{b}} + 0 = (m_{\text{b}} + m_{\text{p}})\vec{v}\,'_{\text{b/p}}$$

$$\vec{v}_{\text{b}} = \frac{(m_{\text{b}} + m_{\text{p}})v'_{\text{b/p}}}{m_{\text{b}}}$$

$$\vec{v}_{\text{b}} = \frac{\left[5.50\ \cancel{\text{g}}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right) + 1.75\ \text{kg}\right]1.566\ \frac{\text{m}}{\text{s}}}{5.50\ \text{g}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)}$$

$$\vec{v}_{\text{b}} = \frac{(1.7555\ \cancel{\text{kg}})1.566\ \frac{\text{m}}{\text{s}}}{0.005\ 50\ \cancel{\text{kg}}}$$

$$\vec{v}_{\text{b}} = 499.8387\ \frac{\text{m}}{\text{s}}$$

$$\vec{v}_{\text{b}} \cong 5.00 \times 10^2\ \frac{\text{m}}{\text{s}}\text{[in positive direction]}$$

The velocity of the bullet just before the collision was about 500 m/s in the positive direction.

Validate the Solution

In both calculations, the units cancelled to give metres per second, which is correct for velocity. The velocity of 500 m/s is a reasonable velocity for a bullet.

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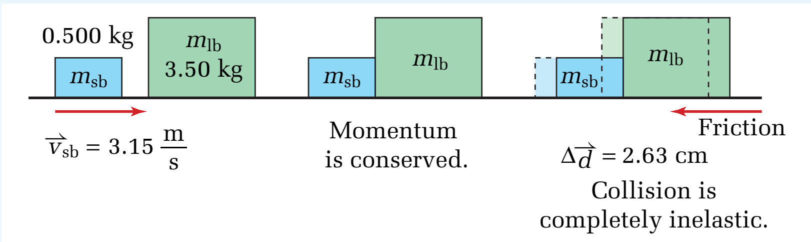
2. A block of wood with a mass of 0.500 kg slides across the floor toward a 3.50 kg block of wood. Just before the collision, the small block is travelling at 3.15 m/s. Because some nails are sticking out of the blocks, the blocks stick together when they collide. Scratch marks on the floor show that they slid 2.63 cm before coming to a stop. What is the coefficient of friction between the wooden blocks and the floor?

Conceptualize the Problem

- Sketch the blocks just before, at the moment of, and after the collision, when they came to a stop.
- *Momentum* is conserved during the collision.



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- Since the blocks stuck together, the collision was *completely inelastic*, so *kinetic energy* was *not* conserved. Some kinetic energy was lost to sound, heat, and deformation of the wood during the collision.
- Some *kinetic energy* remained after the collision.
- The force of *friction* did *work* on the moving blocks, converting the remaining kinetic energy into heat.
- Due to the *law of conservation of energy*, you know that the *work* done by the force of *friction* was equal to the *kinetic energy* of the blocks at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize direction.
- Use the subscripts "sb" for the small block, "lb" for the large block, and "cb" for connected blocks.

Identify the Goal

The coefficient of friction, μ, between the wooden blocks and the floor

Identify the Variables and Constants

Known		Implied	Unknown		
$m_{sb} = 0.500$ kg	$\vec{v}_{sb} = 3.15\ \frac{m}{s}$	$g = 9.81\ \frac{m}{s^2}$	μ	W	$\vec{F}_N$
$m_{lb} = 3.50$ kg	$\Delta\vec{d} = 2.63$ cm	$\vec{v}_{lb} = 0.00\ \frac{m}{s}$	$\vec{F}_f$	E_k	$\vec{v}'_{cb}$

Develop a Strategy

Apply the law of conservation of energy to find the velocity of the connected blocks of wood after the collision.

$$m_{sb}\vec{v}_{sb} + m_{lb}\vec{v}_{lb} = m_{sb}\vec{v}'_{sb} + m_{lb}\vec{v}'_{lb}$$

$$m_{sb}\vec{v}_{sb} + 0 = (m_{sb} + m_{lb})\vec{v}'_{cb}$$

$$\vec{v}'_{cb} = \frac{m_{sb}\vec{v}_{sb}}{m_{sb} + m_{lb}}$$

$$\vec{v}'_{cb} = \frac{(0.500\text{ kg})(3.15\ \frac{m}{s})}{0.500\text{ kg} + 3.50\text{ kg}}$$

$$\vec{v}'_{cb} = \frac{1.575\ \cancel{\text{kg}}\frac{m}{s}}{4.00\ \cancel{\text{kg}}}$$

$$\vec{v}'_{cb} = 0.393\ 75\ \frac{m}{s}\text{[to the right]}$$

Due to the law of conservation of energy, the work done on the blocks by the force of friction is equal to the kinetic energy of the connected blocks after the collision.

$$W_{\text{(to stop blocks)}} = E_{k}\text{ (after collision)}$$

Substitute the expressions for work and kinetic energy into the equations.

$$F_{||}\Delta d = \frac{1}{2}mv^2$$

Friction is the force doing the work, and it is always parallel to the direction of motion. Substitute the formula for the force of friction.

$$F_f\Delta d = \frac{1}{2}mv^2$$

$$\mu F_N\Delta d = \frac{1}{2}mv^2$$

Since the blocks are moving horizontally, the normal force is the weight of the blocks. Substitute the weight into the expression and solve for the coefficient of friction.

$$\mu mg\Delta d = \frac{1}{2}mv^2$$

$$\mu = \frac{\frac{1}{2}\cancel{m}v^2}{\cancel{m}g\Delta d}$$

$$\mu = \frac{v^2}{2g\Delta d}$$

$$\mu = \frac{(0.393\ 75\ \frac{\text{m}}{\text{s}})^2}{2(9.81\ \frac{\text{m}}{\text{s}^2})(2.63\ \cancel{\text{cm}})(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}})}$$

$$\mu = \frac{0.15\ 504\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}{0.5160\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}$$

$$\mu = 0.300\ 46$$

$$\mu \cong 0.300$$

The coefficient of friction between the blocks and the floor is 0.300.

Validate the Solution

All of the units cancel, which is correct because the coefficient of friction is unitless. The value of 0.300 is quite reasonable for a coefficient of friction between wood and another similar surface.

PRACTICE PROBLEMS

18. A 12.5 g bullet is shot into a ballistic pendulum that has a mass of 2.37 kg. The pendulum rises a distance of 9.55 cm above its resting position. What was the speed of the bullet?

19. A student flings a 23 g ball of putty at a 225 g cart sitting on a slanted air track that is 1.5 m long. The track is slanted at an angle of 25° with the horizontal. If the putty is travelling at 4.2 m/s when it hits the cart, will the cart reach the end of the track before it stops and slides back down? Support your answer with calculations.

20. A car with a mass of 1875 kg is travelling along a country road when the driver sees a deer dart out onto the road. The driver slams on the brakes and manages to stop before hitting the deer. The driver of a second car (mass of 2135 kg) is driving too close and does not see the deer. When the driver realizes that the car ahead is stopping, he hits the brakes but is unable to stop. The cars lock together and skid another 4.58 m. All of the motion is along a straight line. If the coefficient of friction between the dry concrete and rubber tires is 0.750, what was the speed of the second car when it hit the stopped car?

21. You and some classmates read that the record for the speed of a pitched baseball is 46.0 m/s. You wanted to know how fast your school's star baseball pitcher could throw. Not having a radar gun, you used the concepts you learned in physics class. You made a pendulum with a rope and a small box lined with a thick layer of soft clay, so that the baseball would stick to the inside of the box. You drew a large protractor on a piece of paper and placed it at the top, so that one student could read the maximum angle of the rope when the pendulum swung up. The rope was 0.955 m long, the box with clay had a mass of 5.64 kg, and the baseball had a mass of 0.350 kg. Your star pitcher pitched a fastball into the box and the student reading

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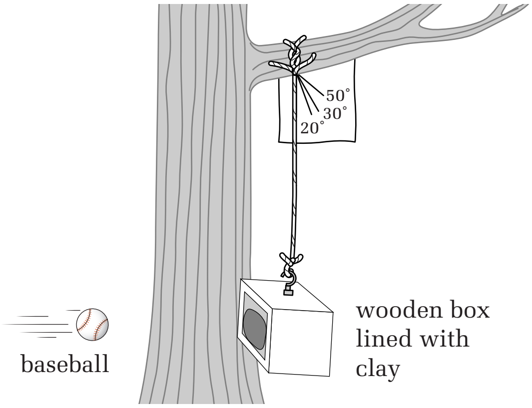
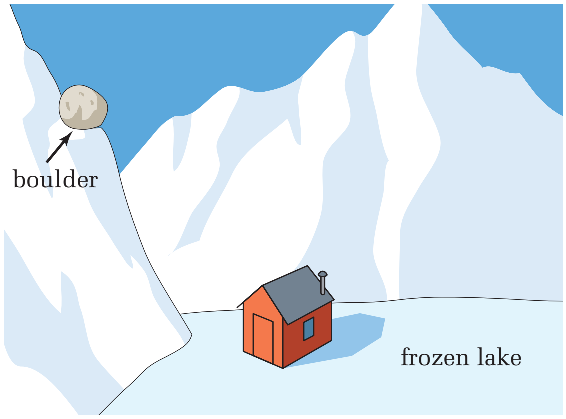
the angle recorded a value of 20.0° from the resting, vertical position. How fast did your star pitcher pitch the ball?



22. A 55.6 kg boulder sat on the side of a mountain beside a lake. The boulder was 14.6 m above the surface of the lake. One winter night, the boulder rolled down the mountain, directly into a 204 kg ice-fishing shack that was sitting on the frozen lake. What was the velocity of the boulder and shack at the instant that they began to slide across the ice? If the coefficient of friction between the shack and the rough ice was 0.392, how far did the shack and boulder slide?



4.3 Section Review

1. K/U What is the difference between an elastic collision and an inelastic collision?

2. C Describe an example of an elastic collision and an example of an inelastic collision that were not discussed in the text.

3. C Given a set of data for a collision, describe a step-by-step procedure that you could use to determine whether the collision was elastic.

4. I The results of the head-on collision in which the moving mass was much larger than the stationary mass ($m_1 >>> m_2$) showed that (a) that the velocity of mass 1 after the collision was almost the same as it had been before the collision and (b) that mass 2, which was stationary before the collision, attained a velocity nearly double that of mass 1 after the collision. Explain how it is possible for kinetic energy ($\frac{1}{2}mv^2$) to be conserved in such a collision, when there was a negligible change in the velocity of mass 1 and a large increase in the velocity of mass 2.

5. MC Imagine that you have a very powerful water pistol. Describe in detail an experiment that you could perform, including the measurements that you would make, to determine the velocity of the water as it leaves the pistol.